Anatomy of an Epidemic

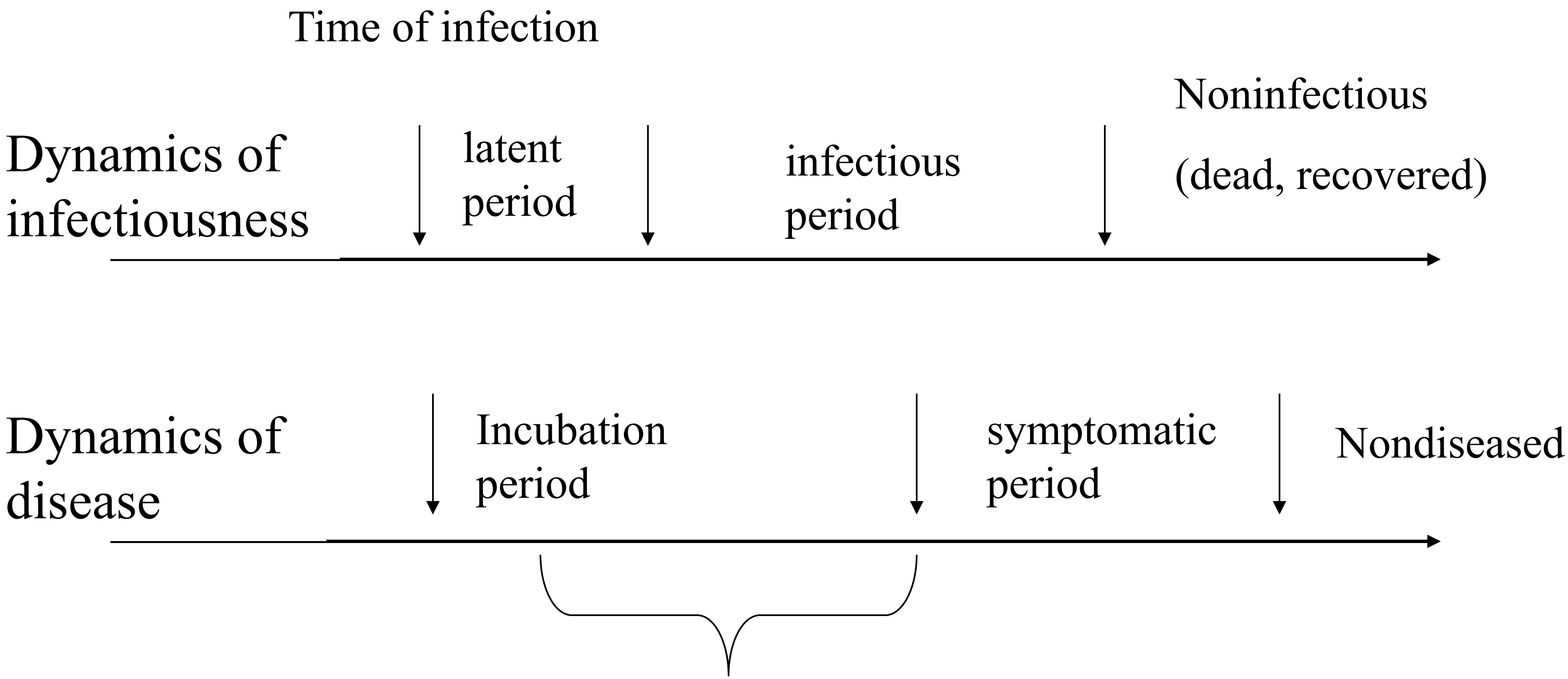
Model of a simple epidemic

- Definitions
  - time periods
  - contact rate
  - transmission parameter
  - susceptibility
  - immunity

Definitions

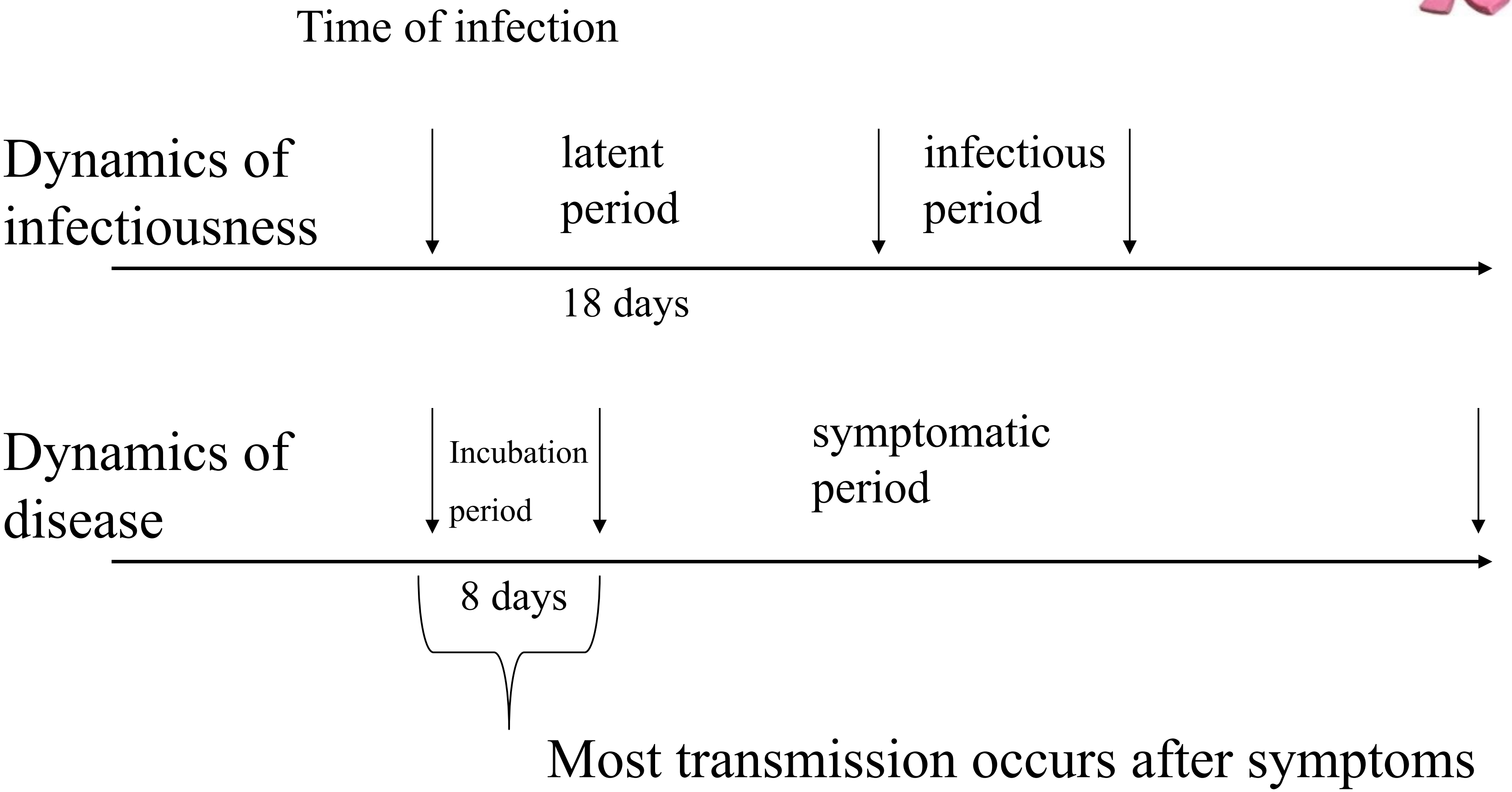
- Incubation period
  - Time before symptoms develop
- Latent period
  - Time from infection until infected person becomes infectious
- Infectious period
  - Time during which infected person is infectious to others
- Symptomatic period
  - Time during which symptoms are present

Time Lines for chickenpox



Most transmission occurs prior to symptoms

Time Lines for Pertussis



Compartmental model of a simple epidemic

- Definitions
  - time periods
  - contact rate
  - transmission parameter
  - susceptibility
  - immunity

Contacts

- Respiratory
  - Cough
  - Sneeze
  - Droplets
  - Talking/singing

Contact rate

k = the average number of relevant contacts made by an infective person per unit time.

Varies by

- age
- sociodemographics
- kind of contact
- employment

- Definitions
  - time periods
  - contact rate
  - transmission parameter
  - susceptibility
  - immunity

Transmission probability

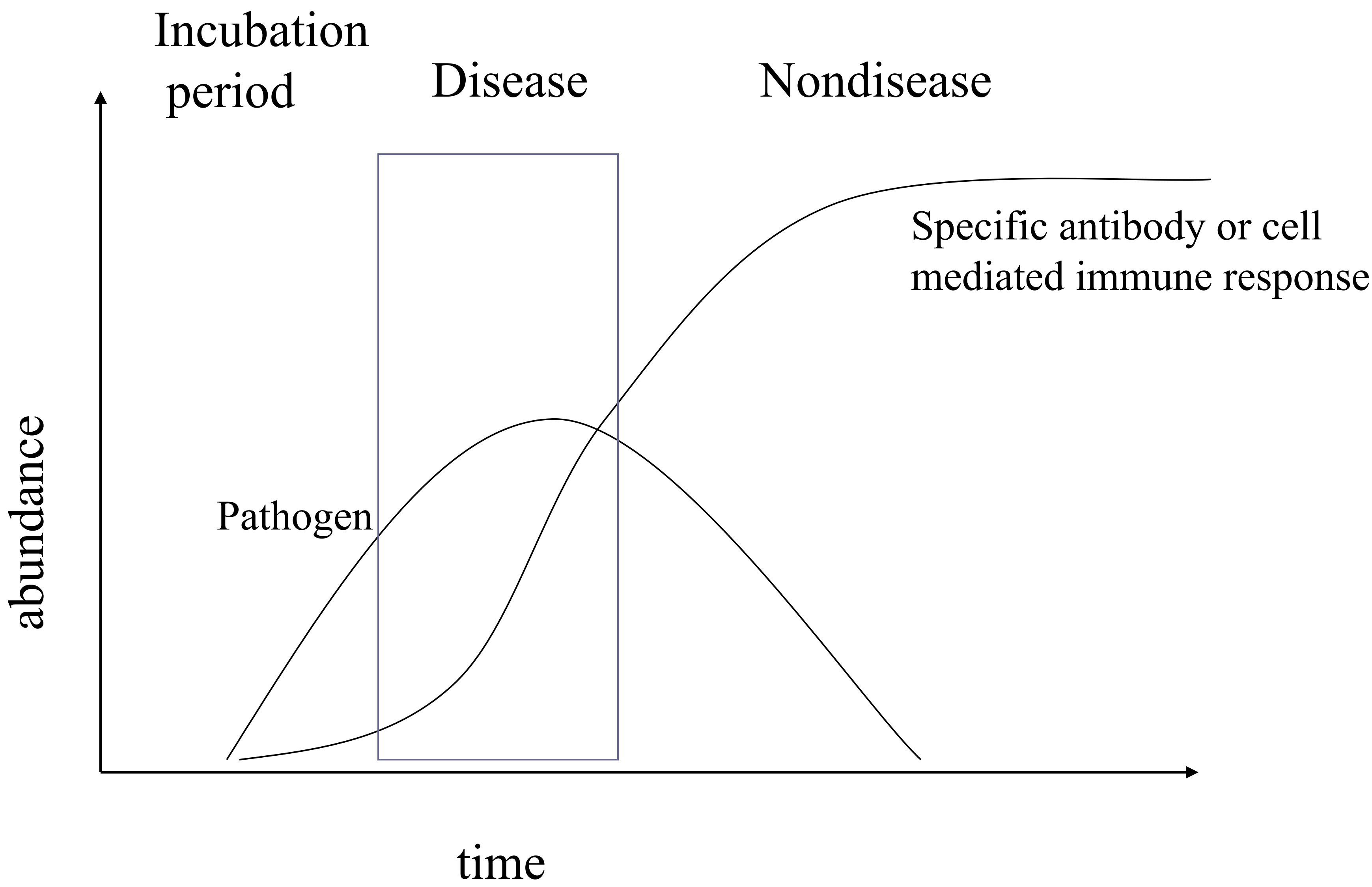
Probability of successful transmission of an infectious disease from an infective to a susceptible person, given contact between that infective and the susceptible person

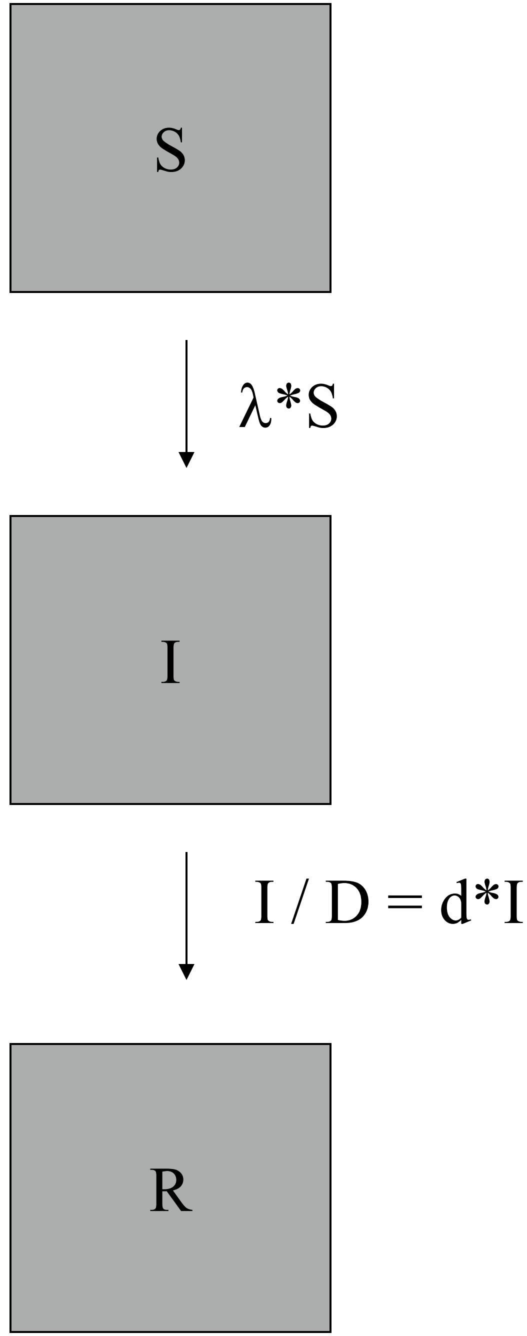
Transmission parameter

- Term combines
  - Transmission probability
  - Contact rate
  - Units are sometime nebulous!

- Definitions
  - time periods
  - contact rate
  - transmission probability
  - susceptibility
  - immunity

Acquired immunity





State Variables:
S=Susceptibles
I=infectious
R=removed

Parameters:

λ = incidence

D=duration of infectiousness

d = recovery rate

Incidence

- The incidence of an infectious disease is a function of
  - contact rate, k
  - transmission probability, *b*
  - prevalence of infectious people

- Theory of dependent happenings
  - number of people affected is dependent on number of people already infected

Incidence rate as a function of prevalence

$\lambda(t) = k*b*Prevalence\ (t)$

Where

$\lambda(t)$ = incidence rate at time t

k=number of people contacted per time unit

b=transmission probability

Consequence

Can use this to estimate $b*k$ (β), which is the transmission parameter.

Since

$\lambda(t) = b*k*Pr(t) = \beta*Pr(t)$

then

$\beta = \lambda(t)/Pr(t)$

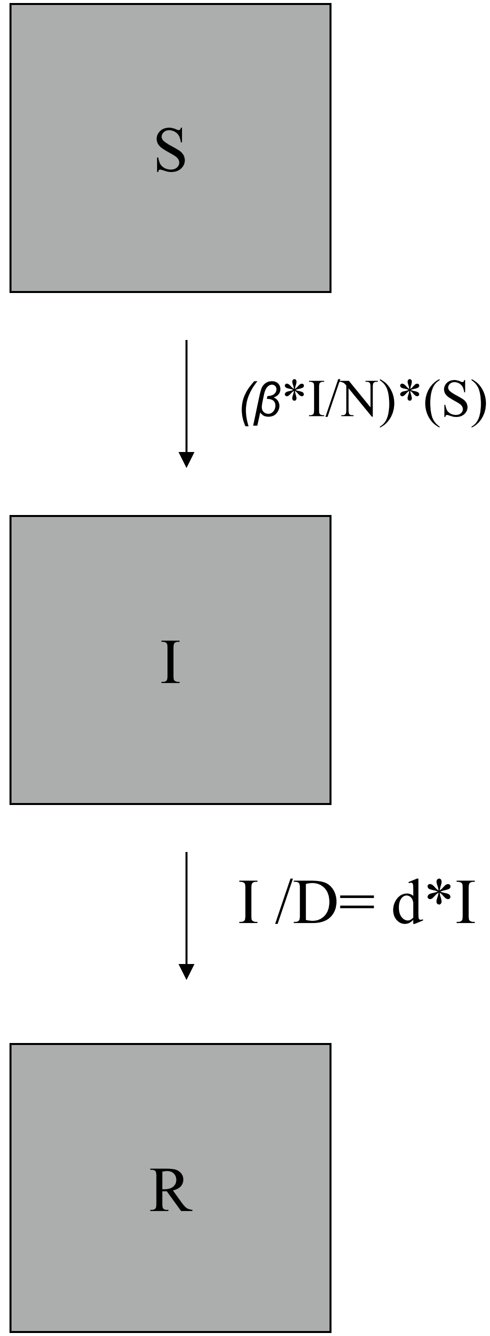
Simple SIR model

<u>State Variables</u>

S=susceptibles

I=infectious

R=removed



<u>Parameters</u>

λ = incidence

D=duration of infectiousness

d=recovery rate

d=1/D

Note that I/N is equivalent to prevalance of cases:

$$Pr(t) = I/N$$

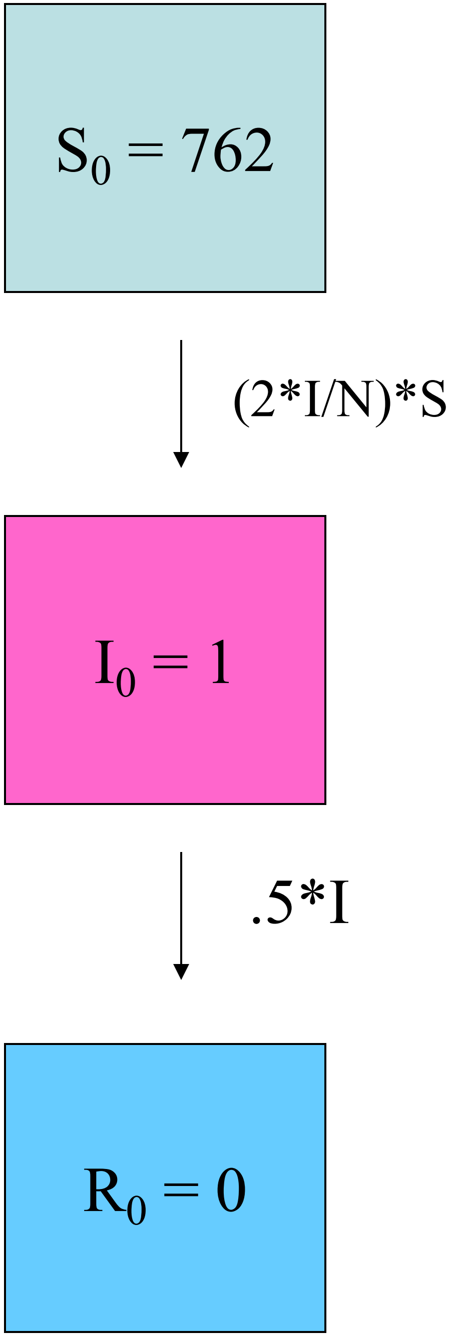
Influenza outbreak in boys' boarding school

- January 1978
- 763 boys returned from winter vacation
- Within 1 week, 1 case of flu
- 2 cases followed within 4-5 days
- By end of month, 50% boys sick
- Most of school affected by mid-Feb
- No further cases after mid-Feb

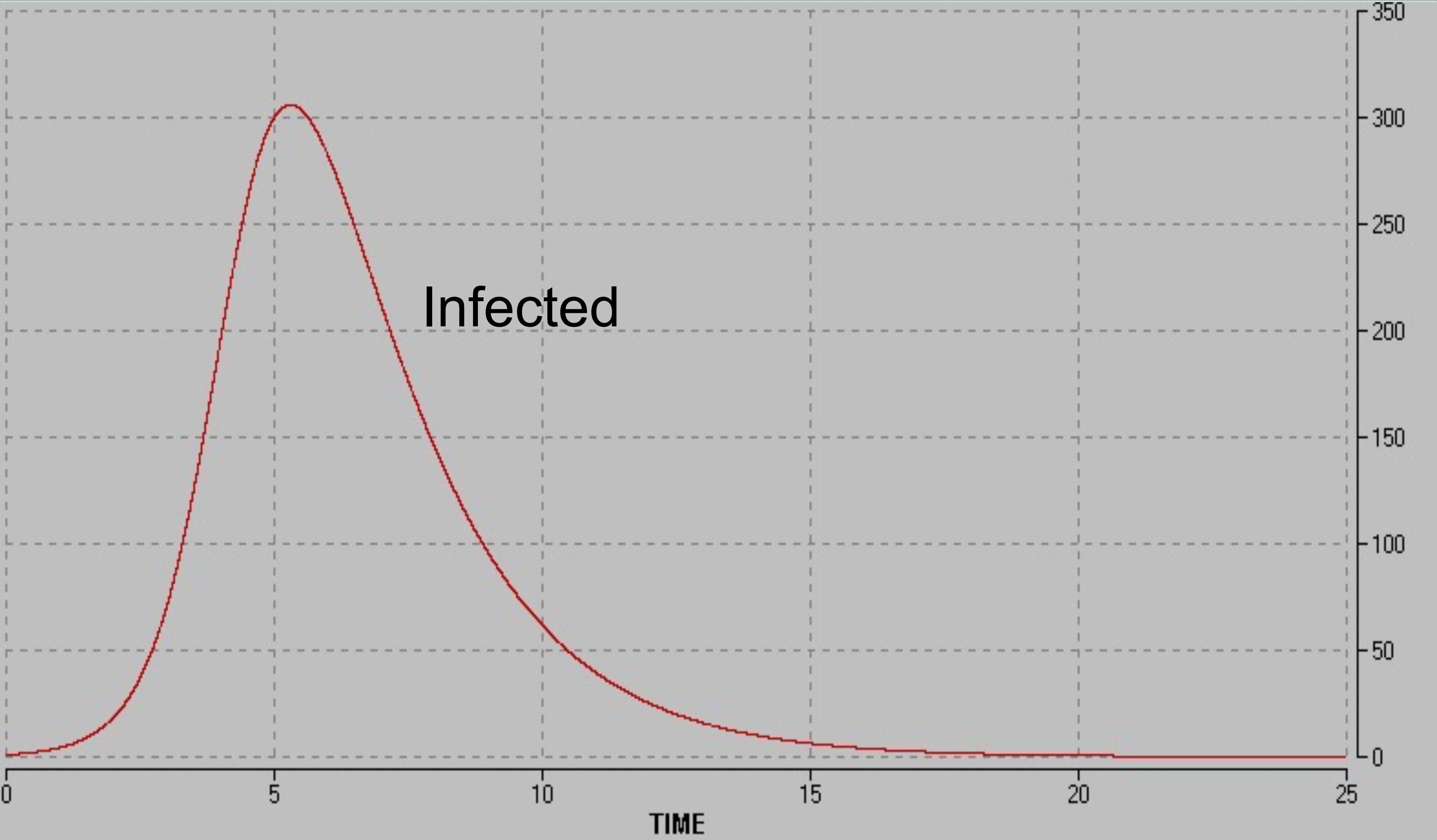
Model

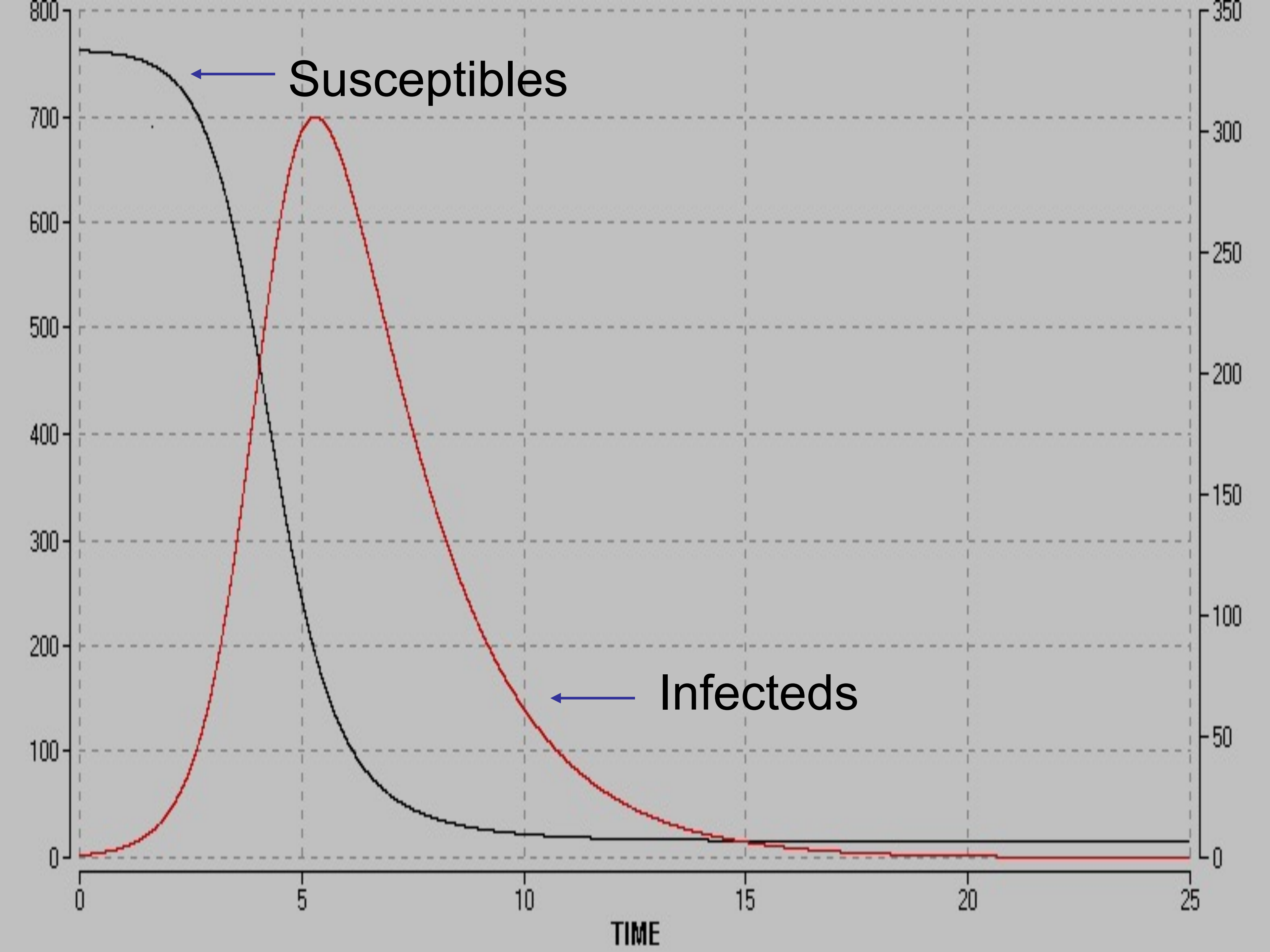
- Initial State Variables
  - S=762
  - I=1
  - R=0
- Parameters
  - D=2 days → d=.5
  - $\beta = 2$

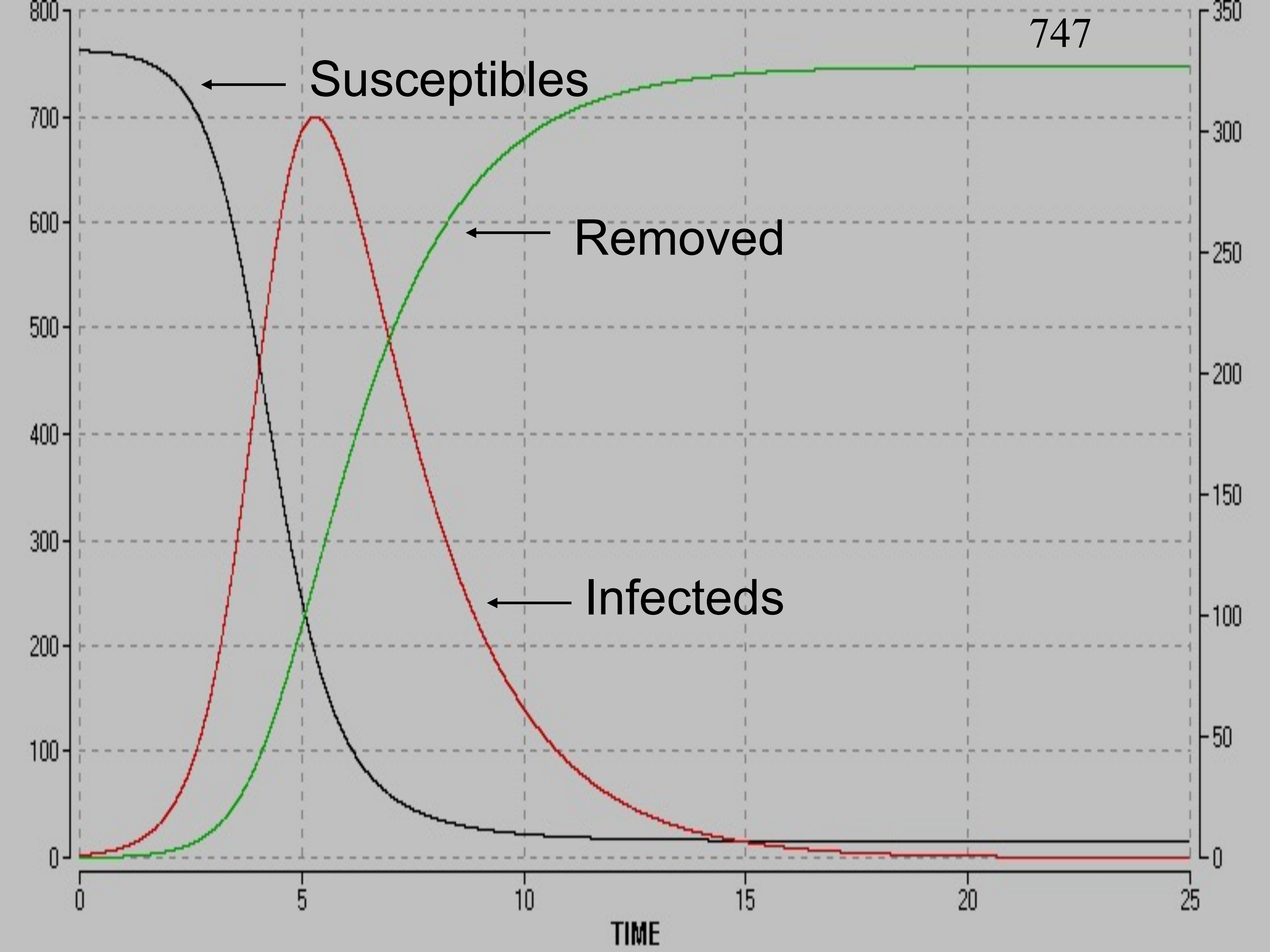
Model of influenza outbreak in boys' boarding school



Infectious boys in an influenza outbreak in boys' boarding school




Susceptibles
Infecteds
800
700
600
500
400
300
200
100
0
350
300
250
200
150
100
50
0
0
5
10
15
20
25
TIME



747
Susceptibles
Removed
Infecteds
TIME
800
700
600
500
400
300
200
100
0
350
300
250
200
150
100
50
0
0
5
10
15
20
25

The basic reproductive number

The number of people who develop an infectious disease as the result of infection by a single infectious case introduced into an entirely susceptible population.

Basic reproductive number

R_0 is a composite function of

- transmission probability
- average number of contacts
- duration of infectiousness

$R_0 = R_0 = b*k/d = \beta*D$

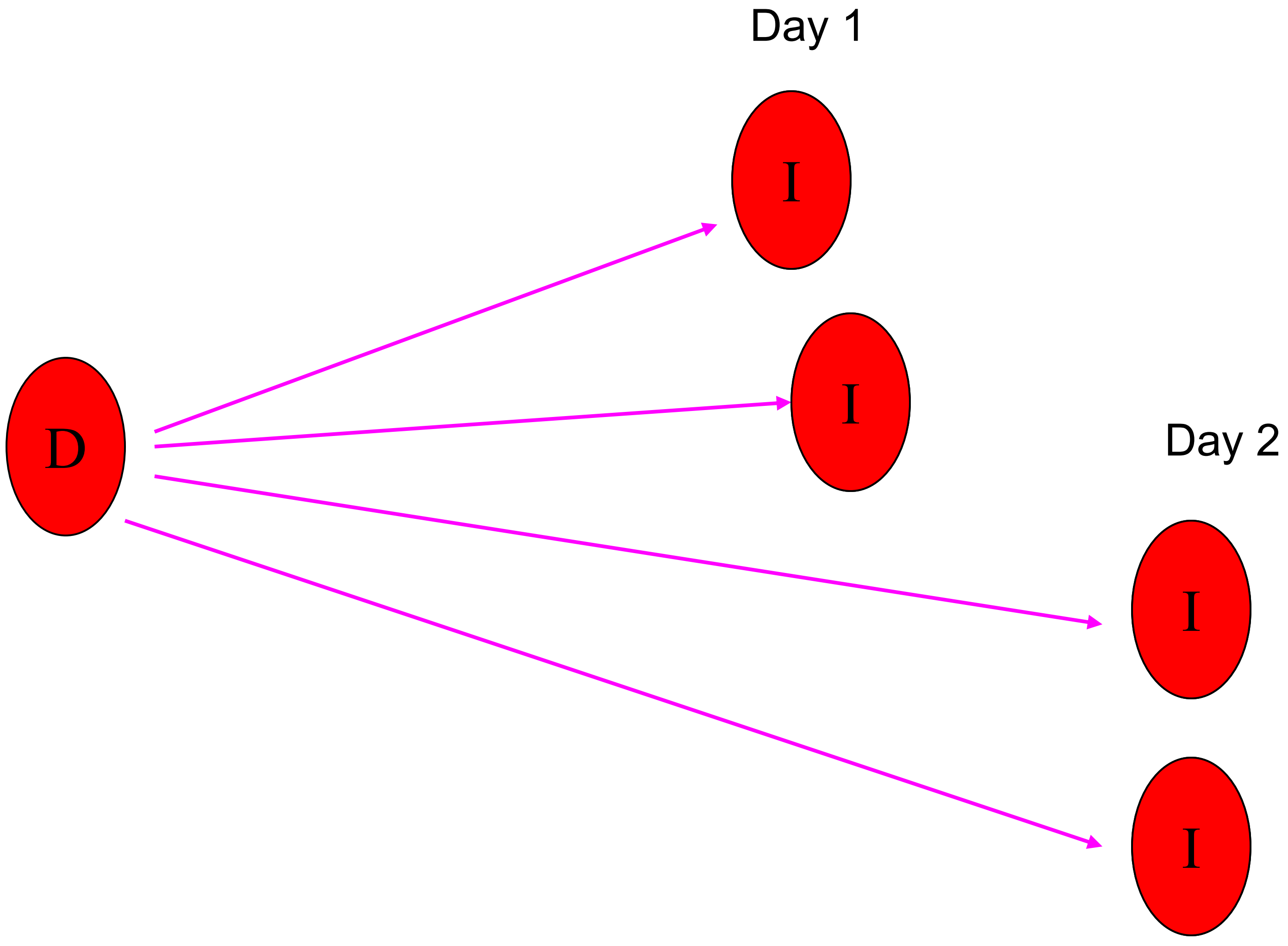
R_0

- If $R_0 > 1$, disease is epidemic
- If $R_0 = 1$, disease is endemic
- If $R_0 < 1$, disease will die out

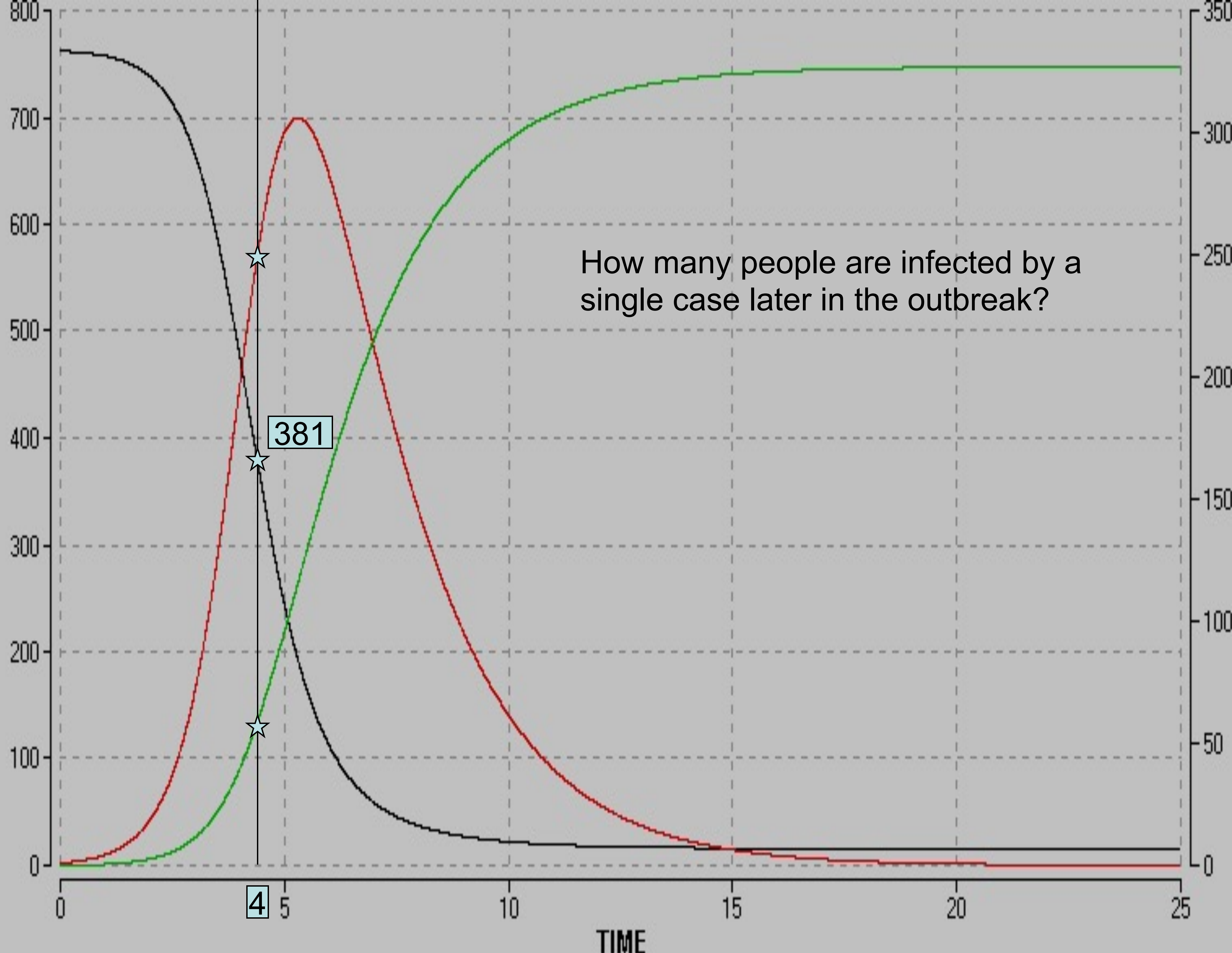
Some R_0s

Measles	England	1947	13-14
	Nigeria	1968	16-17
	Kansas	1920	5-6
Pertussis	England	1944-78	16-18
	Canada	1912	7-8
Chickenpox	USA	1912	7-8
	USA	1944	10-11

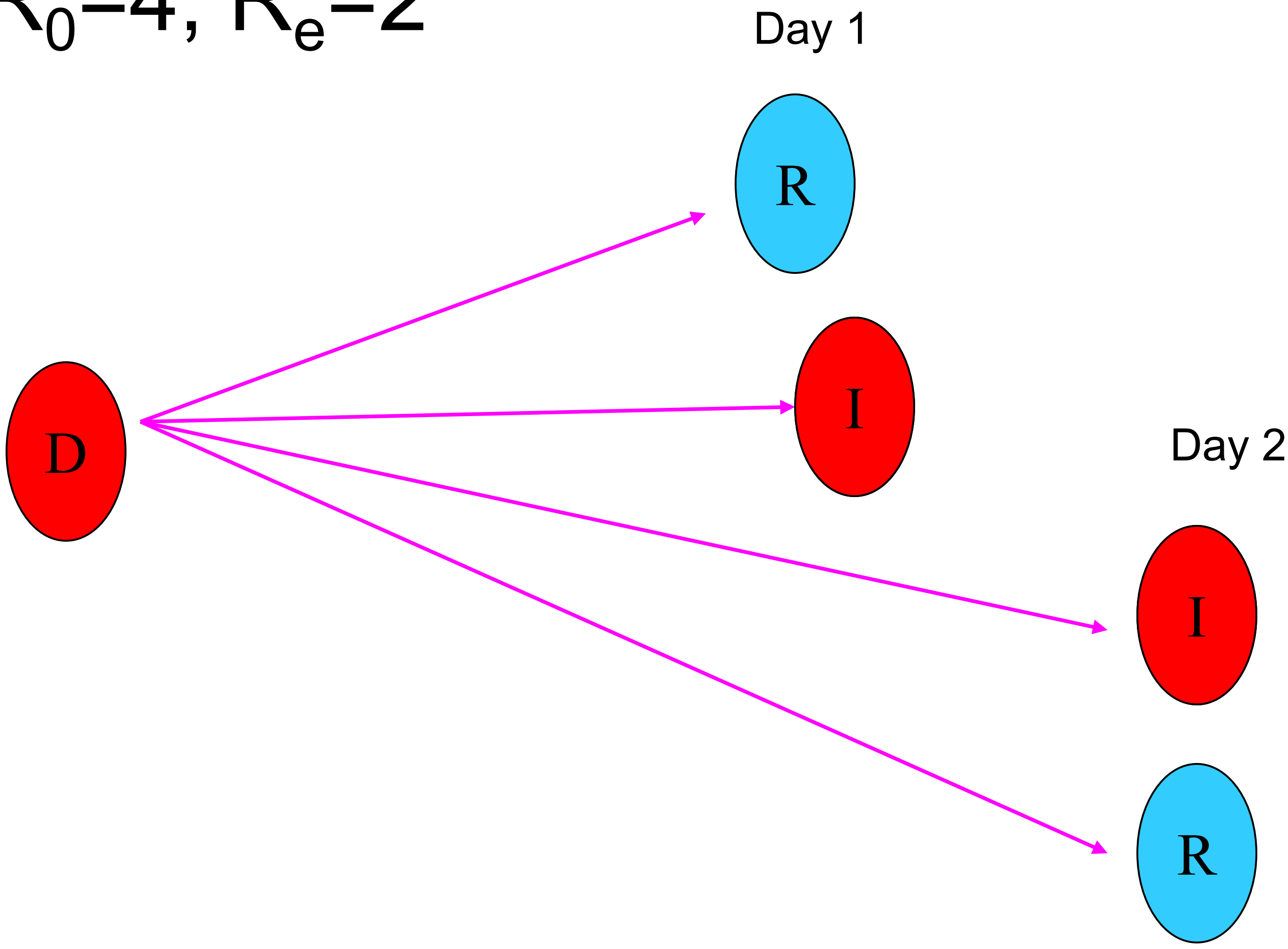
R_0=4



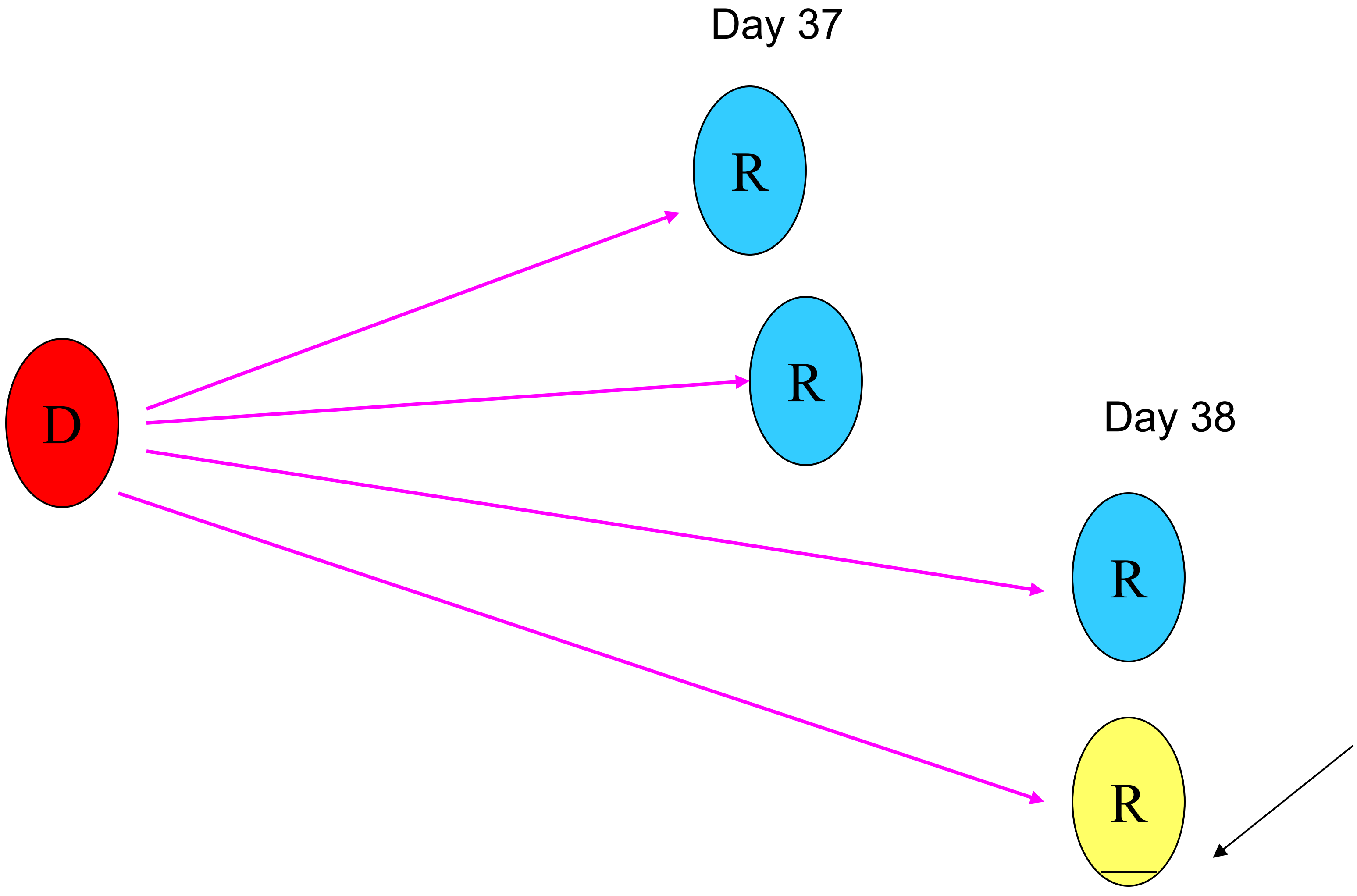
How many people are infected by a single case later in the outbreak?



4 days later
R_0=4, R_e=2



On day 37, 743/762 boys infected; .975 of the population



The effective reproductive number R_E

- Population not entirely susceptible
- Fewer people infected by a single case

- $R_E = R_0 * x$

Where x = proportion of contacts that are with susceptibles

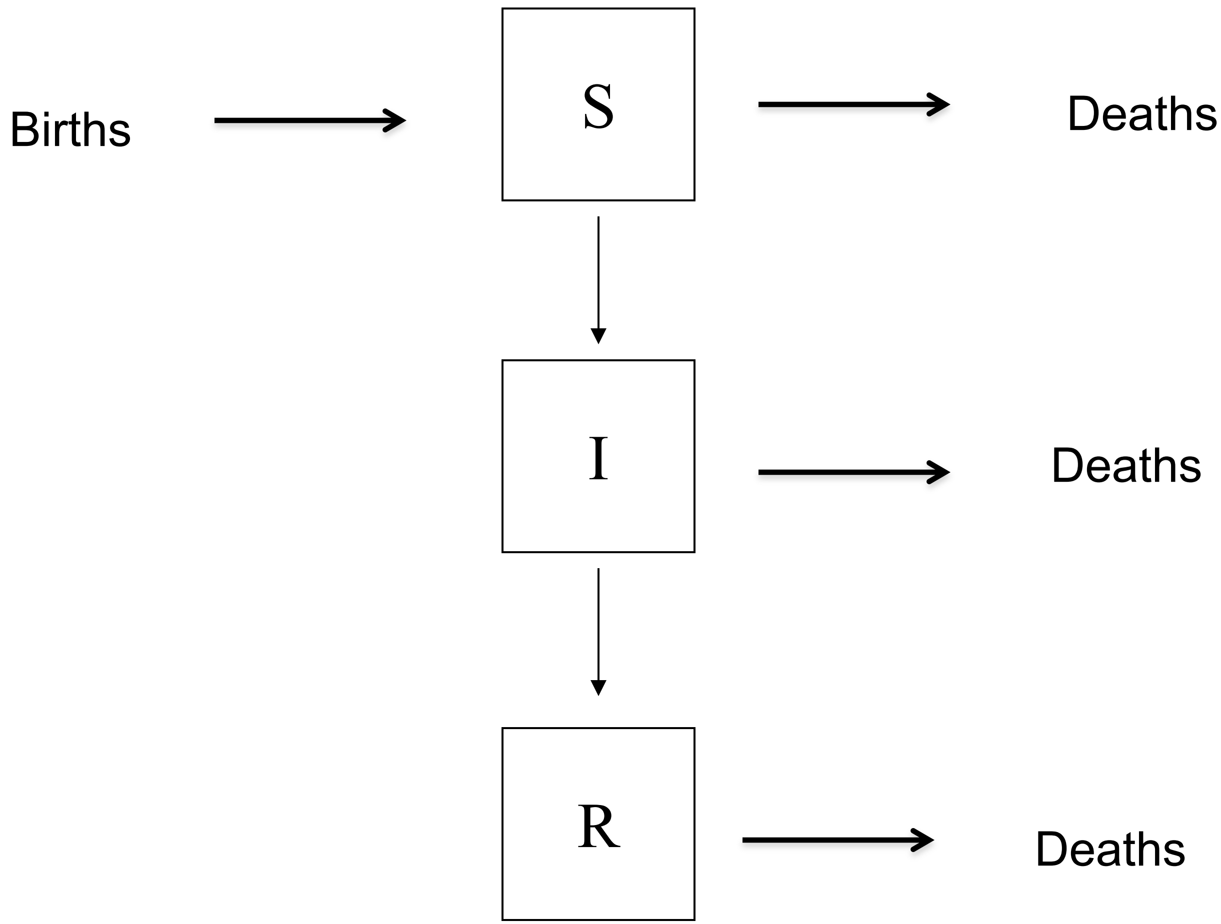
Effective reproductive number

- Not a stable attribute of an infectious disease
- Useful if disease is endemic (or at equilibrium) since R_e=1
- If chickenpox is “endemic” and 20% of the population is susceptible
  - $R_e = R_0 x$ and $R_e = 1$
  - R0 = 1/.2 = 5
- Can use serological data to estimate R_0

Thresholds

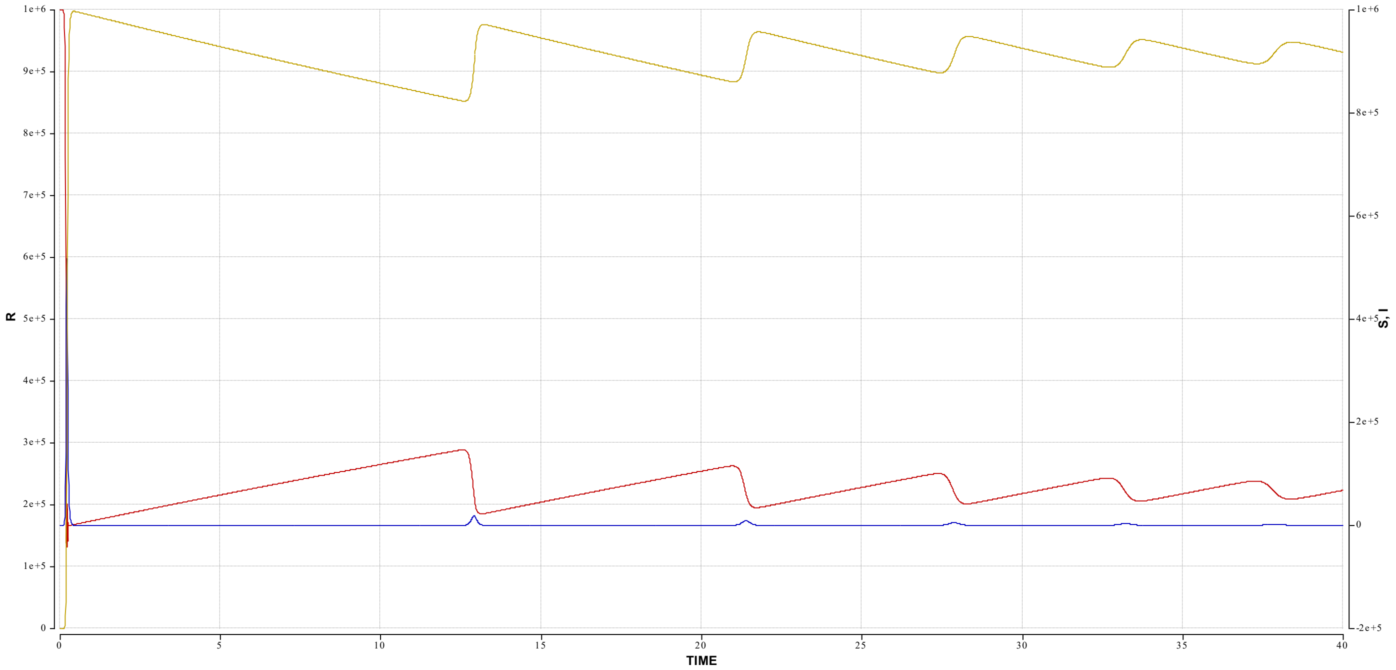
- If initial S > 191, epidemic occurs
- If initial S = 191, endemic disease occurs
- If initial S < 191, no epidemic

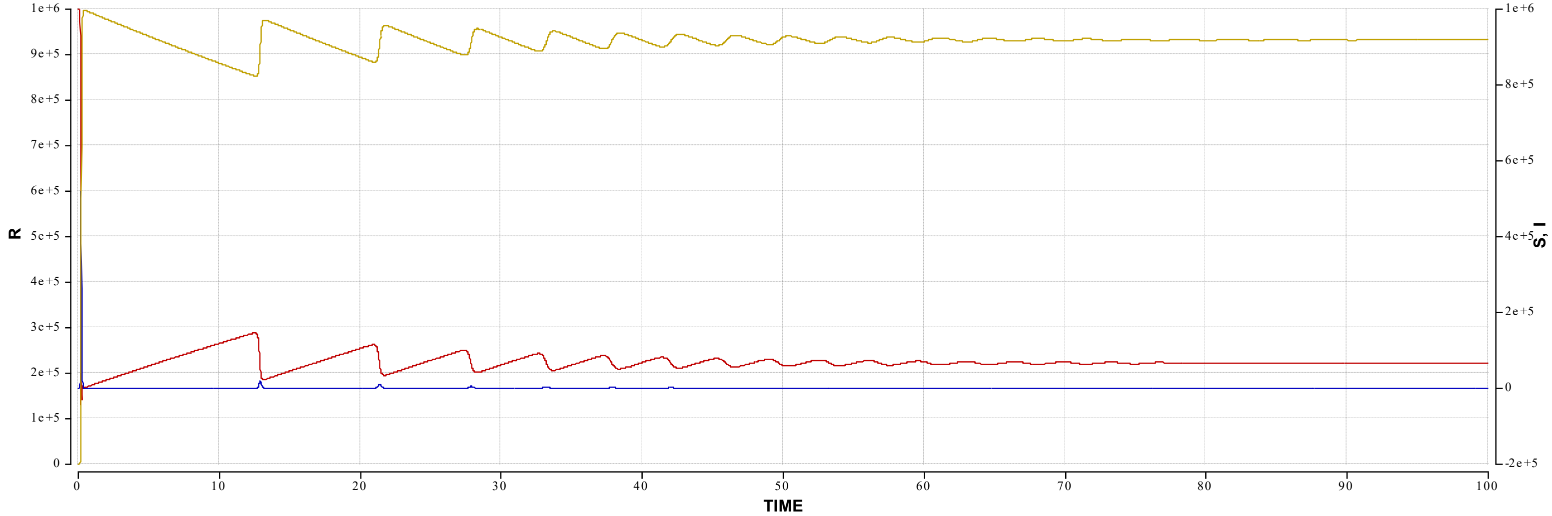
Adding births and deaths



Parameters for measles

- b=.75
- D= 12 days so d = 1/(12/365) = 30
- k= 12 week = 600 year
- For N= 1000000
- For m = 1/72


R
S, I
TIME


R
S, I
TIME

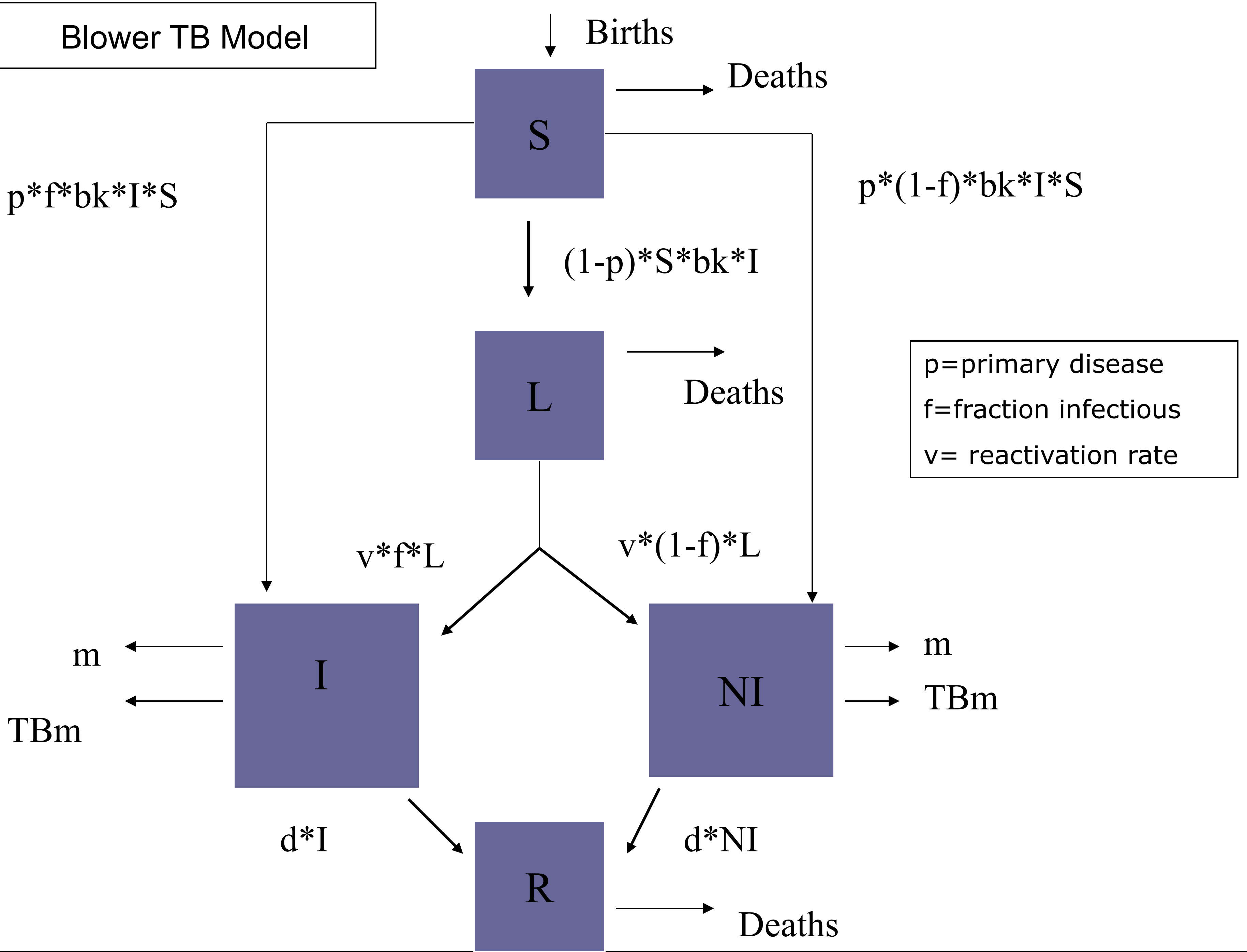
Inferences

- In the absence of other perturbations, the system of differential equations describing the SIR transmission process in an “open” community comes to a stable equilibrium
- The equilibrium value of I varies with R_0
- The equilibrium values of I also depend on the rate of births and deaths in the population.

TB considerations

- Latency
- Reactivation
- Immunity?
  - After infection?
  - After disease?
- Non-infectious disease cases
- Specific interventions

Blower TB Model

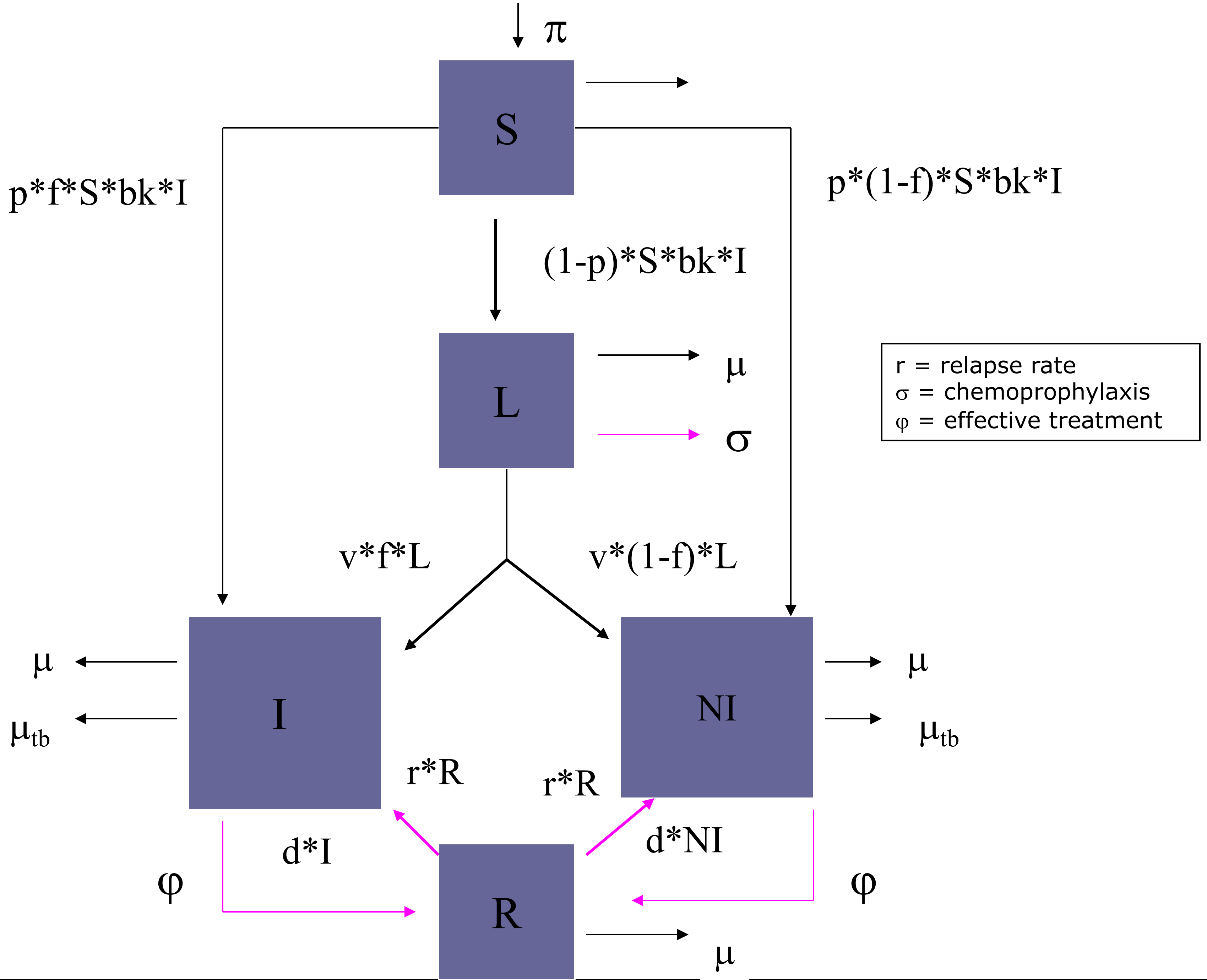


Parameters

Proportion who are fast progressors	p	.05 (0-.3)
Proportion of cases that are infectious	f	.7 (.5-.85)
Reactivation rate	v	.00256-.00527
Transmission parameter	bk	
Recovery rate	d	.058 (.021-.086)
Mortality	m	
TB specific mortality	TBm	.139

Other issues

- Relapse
- Chemoprophylaxis
- Treatment


π
S
p*f*S*bk*I
p*(1-f)*S*bk*I
(1-p)*S*bk*I
L
μ
σ
r = relapse rate
σ = chemoprophylaxis
φ = effective treatment
v*f*L
v*(1-f)*L
μ
μtb
I
NI
μ
μtb
r*R
r*R
d*I
d*NI
φ
φ
R
μ

New parameters

- r = relapse rate = .01-.04
- σ = chemoprophylaxis
- φ = effective treatment

These are variables that depend on treatment programs and goals

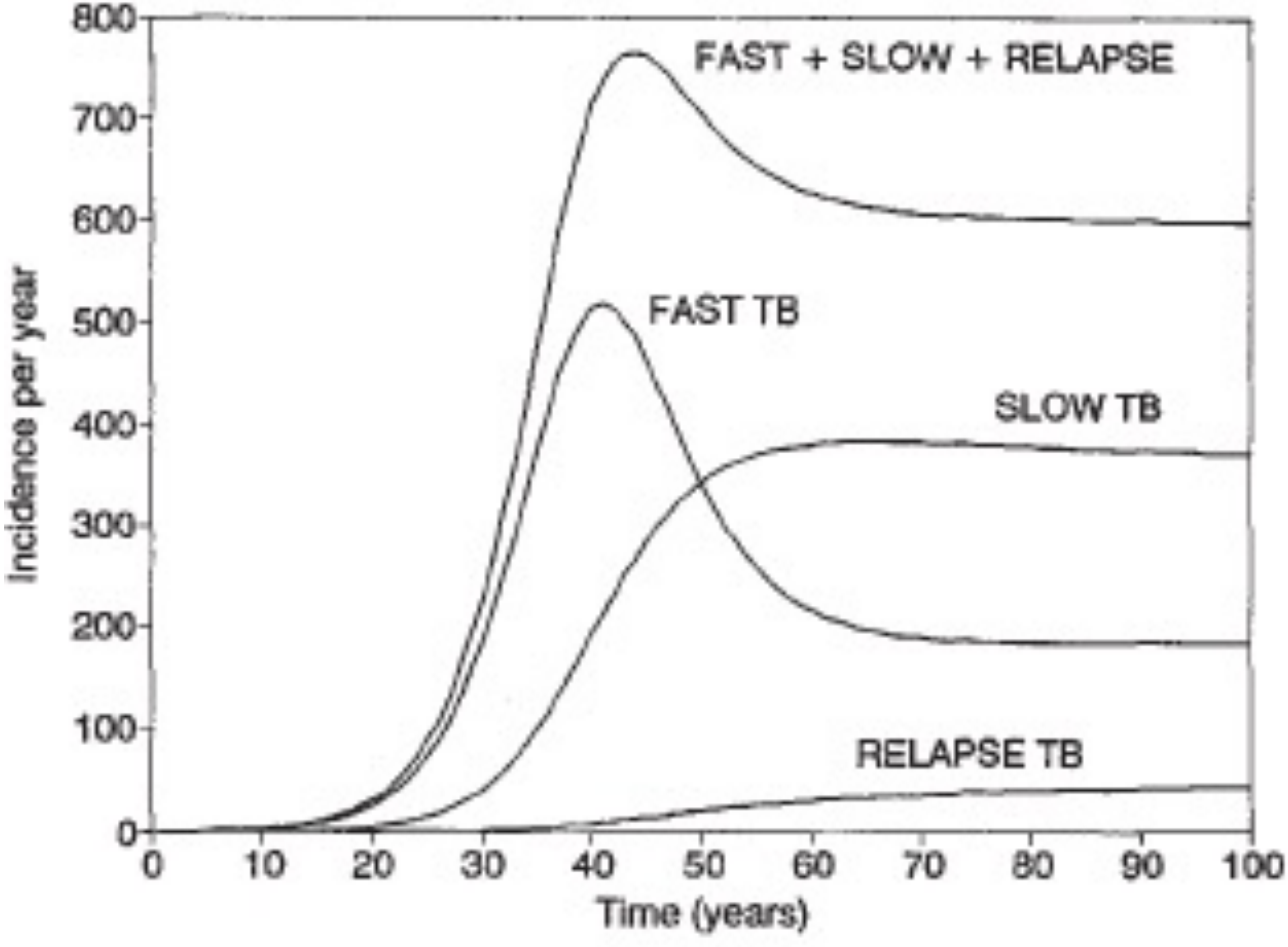
R_0

R_0 for TB = sum of R_0 for slow and fast TB

Parameter values very imprecise

R_0 ranges from .74 - 18.48 with mean of 4.47

Contribution of fast, slow and relapse



How much do you need to do to eradicate disease?

- How much chemoprevention?
- How much case finding and treatment?
- How much of both together?

Levels of treatment required to eradicate TB: combined approaches:

Curves represent eradication thresholds